ECE 486 Control Systems

Spring 2018

Midterm #2 Information

Issued: April 5, 2018
Updated: April 8, 2018

- This document is an info sheet about the *second* exam of ECE 486, Spring 2018.
- Please read the following information carefully and start/continue studying the *second* exam.

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- **When and where is the exam taking place?**

  The second midterm will be held on Thursday, Apr 12, in class from 9:30 a.m. — 10:50 a.m. There is *no conflict exam* offered at any other time.

  For those who cannot make it for legitimate reasons, for each exam that is skipped, their final exam will be reweighted with an additional 15% of the course grade. For example, skipping this exam will make the final count as 40%.

- **What topics will be covered?**

  Everything covered in Lecture 9 through 18 is a fair game. That is everything from Root Locus design to Nyquist Stability Criterion; see lecture matrix for details.

  `https://courses.engr.illinois.edu/ece486/sp2018/lectures/`

  Here is a list of specific topics:

  – All rules for plotting the positive root locus; Positive vs. negative root loci ($0°$ vs. $180°$ root loci); Choosing the gain for pole placement using root locus
  – Design using root locus method: PD and lead control, PI and lag control, lead+lag control
  – Bode magnitude and phase plots
  – Bode plots and stability, gain and phase margins; Bodes gain-phase relationship (as design guideline, not the exact integral relationship)
  – Relations between properties of the open-loop Bode plot (phase margin, crossover frequency) and properties of the closed-loop system (damping ratio, overshoot, bandwidth)
  – Frequency response design method: PD and lead control, PI and lag control, lead+lag control

  - Nyquist plots and Nyquist stability criterion (proof via argument principle not tested on the exam)

- **What to bring during the exam?**

  The exam is closed-book, closed-notes. You may bring one sheet (double-sided, letter size 8.5×11 inch) of notes with any necessary formulas. A simple calculator without symbolic computation is allowed.

- **Any tips for studying the exam?**

  The primary goal of the exam is to test your understanding of the main concepts, not memorization or computational skills. Make sure you can follow all the lecture material, readings, and homework problems and solutions. On the next page, an exam from a past semester is given as a sample. An outline of solutions to this sample exam ~~will be posted~~ is posted alongside the sample exam ~~on the course website this weekend before our exam~~.

  Disclaimer: The exam this semester will be significantly different in style and content from that older one.

- **Is there any extra office hours?**

  There is a one-hour session of extra office hours on Wednesday, April 11, 12 p.m. — 1 p.m. in 3034 ECEB on top of the normal office hours the following hour at the same location.

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1. **Problem 1**: Consider the following transfer function

$$G(s) = \frac{1}{(s-1)(s+3)(s+5)}.$$

Fill in the information below and sketch the root locus of $1 + KG(s)$.

(a) Angles of asymptotes __________

(b) Origin of asymptotes __________

(c) Range of K for stability __________

Solution: Let n be the degree of the denominator of $G(s)$ and m the degree of the numerator of $G(s)$. We have $n = 3$, $m = 0$.

(a) From `lec11.html`, by Rule E, the angles of asymptotes are given by

$$\begin{aligned}\angle s &\approx \frac{180^\circ + \ell \cdot 360^\circ}{n-m} \\ &= \frac{(2\ell+1)\cdot 180^\circ}{n-m}, \qquad \ell = 0, 1, \ldots, n-m-1.\end{aligned}$$

So the angles are 60°, 180° and 300°.

(b) By the additional notes on root locus at the beginning of `lec12.html`, the origin of asymptotes is given by

$$\alpha = \frac{\sum_{i=1}^{n} p_i - \sum_{j=1}^{m} z_j}{n-m}.$$

Therefore the origin is $\frac{1 + (-3) + (-5)}{3 - 0} = -\frac{7}{3}$.

(c) The characteristic polynomial is

$$(s-1)(s+3)(s+5) + K = s^3 + 7s^2 + 7s + K - 15.$$

By Example 3 from `lec07.html`, for a third order polynomial to be stable, we need

$$\begin{cases} 7 > 0, \\ K - 15 > 0, \\ 7 \cdot 7 > K - 15. \end{cases}$$

Hence $15 < K < 64$.

Better use an alternative here. Evaluate the characteristic polynomial at $s = j\omega$ to get the $j\omega$-crossing and the critical K at the same time, i.e.,

$$\begin{aligned} &(j\omega)^3 + 7(j\omega)^2 + 7(j\omega) + K - 15 = 0 \\ \Longrightarrow\ &(-7\omega^2 + K - 15) + j(7\omega - \omega^3) = 0. \end{aligned}$$

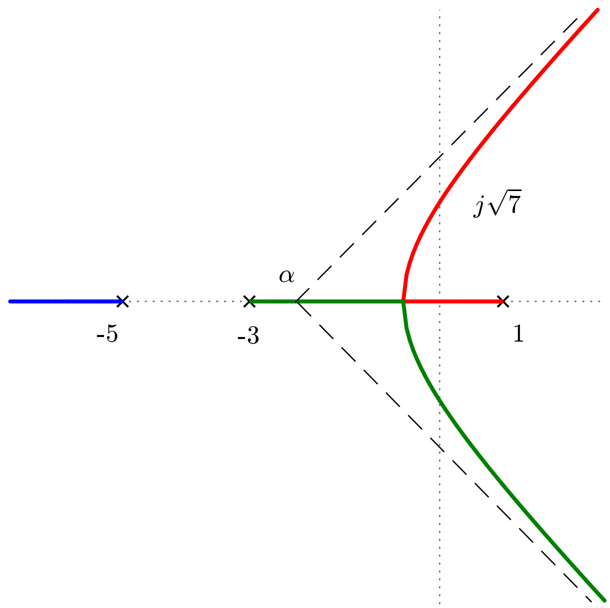
Again $K_{\text{critical}} = 64$ and root locus crosses $\pm j\omega = \pm j\sqrt{7}$ on the imaginary axis.

(d) On top of the three pieces of information above already calculated, we need to decide the real locus and breakaway point.

i. The real locus is given by $(-\infty, -5] \cup [-3, 1]$ by testing the total number of open-loop zeros and poles to the right of a point on the real axis. If the total number of zeros and poles is odd, then the point belongs to the real locus.

ii. The breakaway point is obtained by solving $\frac{\mathrm{d}G(s)}{\mathrm{d}s} = 0$, which gives $s = -0.569$.

The root locus sketch of $1 + KG(s)$ is shown below.



2. **Problem 2**: Sketch the asymptotic Bode **magnitude** plots for the following transfer functions.

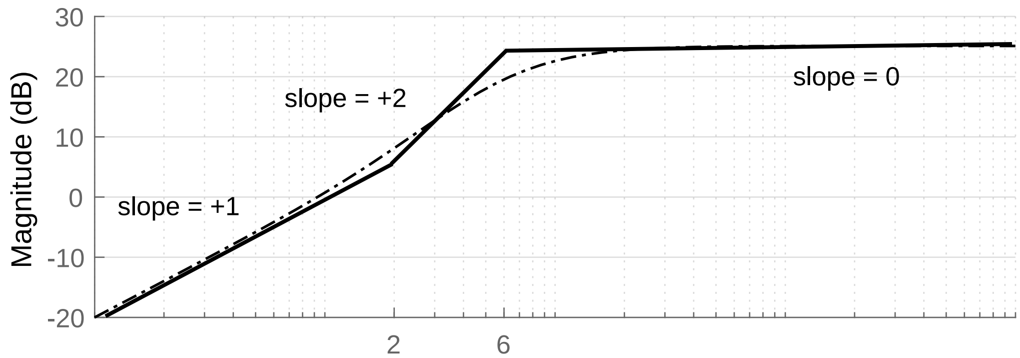
(a) $G(s) = \frac{18s(s+2)}{(s+6)^2}$.

Solution: Write $G_c(s)$ in Bode form,

$$G_c(s) = 18 \cdot 2 \cdot \frac{1}{36} \cdot s(\frac{s}{2}+1)\frac{1}{\left(\frac{s}{6}+1\right)^2}$$

$$= s \cdot (\frac{s}{2}+1) \cdot \frac{1}{\left(\frac{s}{6}+1\right)^2}.$$

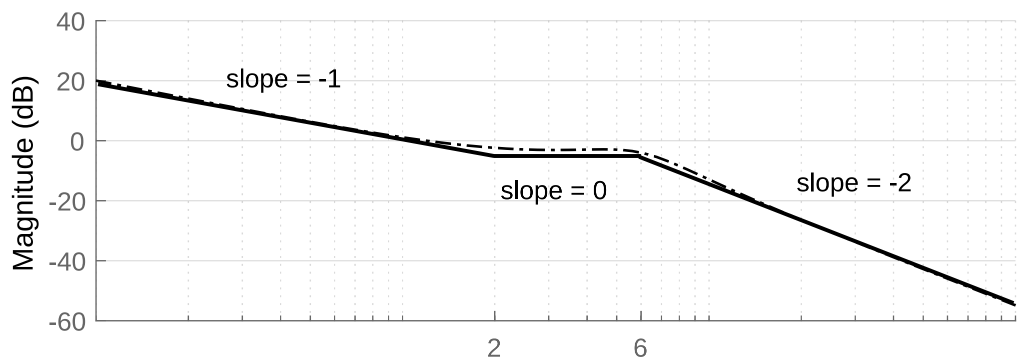
Giving straight lines as the asymptotic indicators is enough.



(b) $G(s) = \dfrac{18(s+2)}{s(s^2+5s+36)}$.

Solution: Similar to (a), use straight lines as Bode sketch for $G_c(s)$ in Bode form,

$$G_c(s) = \frac{1}{s}\cdot(\frac{s}{2}+1)\cdot\frac{1}{\left(\frac{s}{6}\right)^2+\frac{5}{6}\left(\frac{s}{6}\right)+1}.$$



3. **Problem 3**: Consider the unity *negative* feedback loop with plant

$$G_p(s) = \frac{1}{s(s+2)}.$$

Design a Lead Compensator $G_c(s)$ such that the closed-loop system has a pair of poles at $s=-4$.

Solution: Consider a Lead Compensator of the form $G_c(s) = K\dfrac{s+z}{s+p}$. We can use $s+z = s+2$ to cancel out the open-loop pole $s=-2$ so that the resulting closed-loop system is second order.

Then the characteristic equation becomes

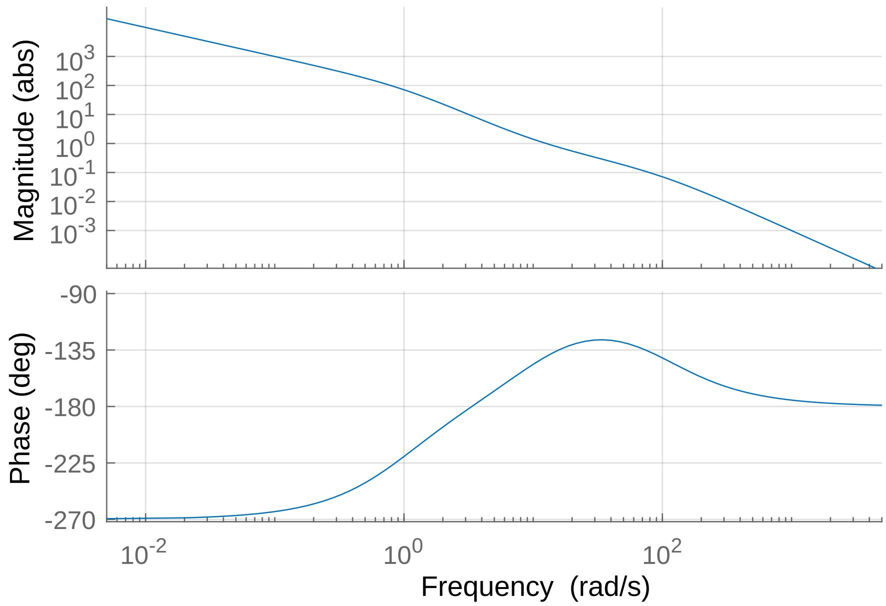
$$1 + KG_c(s)G_p(s) = 1 + K\frac{1}{s(s+p)}.$$

The resulting characteristic polynomial is s^2+ps+K. Matching the desired characteristic polynomial $(s+4)^2 = s^2+8s+16$, we obtain

$$\begin{cases} p &= 8, \\ K &= 16. \end{cases}$$

So one possible Lead Compensator is $G_c(s) = 16\dfrac{s+2}{s+8}$.

4. **Problem 4**: Consider the given Bode plot of some open-loop transfer function $G(s)$. $G(s)$ does not have repeated poles at origin.



Based on the figure, answer the following questions.

(a) Why does the phase plot start out at $-270°$ at low frequencies? Circle one of the possible reasons from below.

i. $G(s)$ has a pole at $s = -270$.

ii. $G(s)$ has a zero at $s = -270$.

iii. $G(s)$ has one RHP open-loop pole.

iv. The phase margin is $270°$.

Solution: $G(s)$ does not have repeat poles at origin, but $G(s)$ may have a **simple** pole at origin $s = 0$. This factor $\frac{1}{s}$ contributes phase $-90°$ initially at low frequencies. Further notice that a factor associated with one RHP open-loop pole contributes phase $-180°$ at low frequencies. Therefore the transfer function has the form

$$G(s) = \frac{1}{s(s-p)} * \cdots ,$$

where $s = p$ is an unstable open-loop pole.

(b) If we enclose the open-loop transfer function $G(s)$ with negative unity feedback with scalar gain $K = 1$, based on the given Bode plot, is this closed-loop system stable? Why or why not?

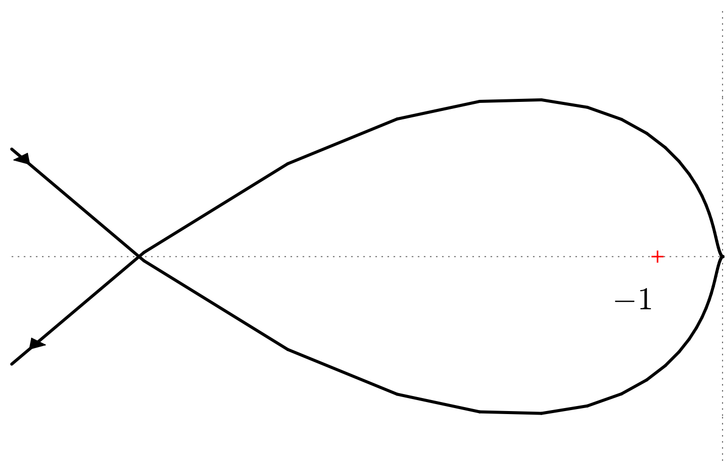
Solution: With $K = 1$, the Bode plot of the equivalent open-loop transfer function (forward path) $KG(s)$ is the same as the Bode plot of $G(s)$. By the given Bode plot of $G(s)$, the phase at crossover frequency is between $-135°$ and $-180°$, hence a positive phase margin (PM $> 0°$). The closed-loop system is stable.

(c) As a follow-up question to (b), if we lower K to $\frac{1}{100}$, is this closed-loop system stable? Why or why not?

Solution: Similar to (b), read the phase of $G(s)$ at the frequency where the magnitude is 100, the phase is between $-225°$ and $-180°$. Hence the PM of $KG(s) = \frac{1}{100}G(s)$ is less than $0°$. The closed-loop system is not stable.

(d) (Bonus question) Sketch the Nyquist plot of $G(s)$ based on its given Bode plot, then answer the question in (b) using Nyquist stability criterion and your answer in (a). Justify your answer.

Solution: Based on the Bode plot of $G(s)$, the magnitude keeps shrinking as ω increases from 0^+ to $+\infty$. By the phase plot, it crosses the negative portion of real axis once. Most importantly, when it crosses the real axis, the magnitude is greater than 1. The crossing is on the left of -1.



It ends at origin with arriving angle $-180°$. One **possible** Nyquist plot (generated using $G(s) = \frac{1}{s(s-1)}100\frac{\frac{s}{10}+1}{\frac{s}{100}+1}$) is shown above.

By (a), there is one RHP open-loop pole, so $P = 1$. Encirclement of Nyquist plot of $G(s)$ about -1 is 1 anticlockwise, so $N = -1$. Then by the Nyquist Stability Criterion from `lec18.html`, the number of RHP closed-loop poles is

$$\begin{aligned} Z &= N + P \\ &= 0. \end{aligned}$$

The closed-loop system is stable, the same as in (b).